V1.1. Linear algebra. List of problems.

V1.1.1. Solve the following homogeneous system of linear equations. Determine a fundamental system of solutions.

$$\begin{cases} 2x_1 + x_2 + 3x_3 - 4x_4 - 3x_5 = 0 \\ x_1 - 2x_2 - 3x_3 + x_4 - 2x_5 = 0 \\ 3x_1 - 3x_2 + 7x_3 + 10x_4 - 16x_5 = 0 \end{cases}$$

V1.1.2. Determine the general solution of the following non-homogeneous system of linear equations.

$$\begin{cases} 2x_1 + x_2 - 3x_3 - 4x_4 &= 6 \\ x_1 - 2x_2 - 3x_3 + 7x_4 &= -10 \\ 2x_1 - x_2 + x_3 - 2x_4 &= -4 \\ x_1 - x_2 + x_3 + 2x_4 &= 1 \end{cases}$$

V1.1.3. Determine the characteristic polynomial, the eigenvalues and the eigenvectors of the following matrix A. Write down the corresponding diagonal matrix D and a transition matrix C.

$$A = \begin{pmatrix} -48 & 20 & 10 \\ -129 & 54 & 27 \\ 37 & -16 & -9 \end{pmatrix} .$$

V1.1.4. Factor the characteristic polynomial over $\mathbb{C}$ and determine the Jordan form of the following matrix A.

$$A = \begin{pmatrix} -46 & -14 & -13 & -8 \\ -160 & -40 & -38 & -31 \\ 257 & 72 & 68 & 46 \\ 146 & 43 & 40 & 26 \end{pmatrix} .$$

V1.1.5. Determine an orthogonal basis of a subspace $L \subset \mathbb{R}^4$ spanned by the following vectors

$$V_1 = (1, 2, 2, -1),\ V_2 = (1, 1, -5, 3),\ V_3 = (3, 2, 8, -7) .$$

V1.1.6. Determine the distance between the following vector X (the endpoint of X) and a subspace $L \subset \mathbb{R}^4$ given as the subspace of solutions of the following homogeneous linear system:

$$X = (4, 4, -4, 0) , \quad L : \begin{cases} x_1 + 2x_2 + 2x_3 - x_4 = 0 \\ x_1 + x_2 - 5x_3 + 3x_4 = 0 . \end{cases}$$

V1.1.7. Determine $\exp(tA)$ where A is a given matrix, $t \in \mathbb{R}$.

$$A = \begin{pmatrix} -1 & 3 \\ 1 & 1 \end{pmatrix} .$$

V1.2. Linear algebra. List of problems.

V1.2.1. Solve the following homogeneous system of linear equations. Determine a fundamental system of solutions.

$$\begin{cases} 7x_1 + 2x_2 + \ \ x_3 - 15x_4 + 10x_5 = 0 \\ \ \ x_1 + 3x_2 + \ \ x_3 - \ \ 6x_4 - \ \ 9x_5 = 0 \\ 2x_1 - 2x_2 \qquad\quad + \ \ 2x_4 + 10x_5 = 0 \end{cases}$$

V1.2.2. Determine the general solution of the following non-homogeneous system of linear equations.

$$\begin{cases} 3x_1 - \ \ x_2 - 8x_3 - 4x_4 &= \ \ 0 \\ \ \ x_1 - 2x_2 - 5x_3 - 4x_4 &= -6 \\ \ \ x_1 + 4x_2 + 3x_3 + 5x_4 &= \ 16 \\ 2x_1 - 3x_2 + 5x_3 + 5x_4 &= \ \ 3 \end{cases}$$

V1.2.3. Determine the characteristic polynomial, the eigenvalues and the eigenvectors of the following matrix A. Write down the corresponding diagonal matrix D and a transition matrix C.

$$A = \begin{pmatrix} -12 & -5 & 0 \\ 35 & 10 & -2 \\ -74 & -24 & 3 \end{pmatrix} .$$

V1.2.4. Factor the characteristic polynomial over $\mathbb{C}$ and determine the Jordan form of the following matrix A.

$$A = \begin{pmatrix} 66 & -67 & -76 & 71 \\ 225 & -233 & -264 & 248 \\ 91 & -95 & -104 & 100 \\ 249 & -260 & -291 & 276 \end{pmatrix} .$$

V1.2.5. Determine an orthogonal basis of a subspace $L \subset \mathbb{R}^4$ spanned by the following vectors

$$V_1 = (-2, 1, 1, -1), \ V_2 = (-3, 5, 8, -2), \ V_3 = (8, 3, 9, 3) .$$

V1.2.6. Determine the distance between the following vector X (the endpoint of X) and a subspace $L \subset \mathbb{R}^4$ given as the subspace of solutions of the following homogeneous linear system:

$$X = (3, 2, 5, 1) , \quad L : \begin{cases} -2x_1 + \ \ x_2 + \ \ x_3 - \ \ x_4 = 0 \\ -3x_1 + 5x_2 + 8x_3 - 2x_4 = 0 . \end{cases}$$

V1.2.7. Determine $\exp(tA)$ where A is a given matrix, $t \in \mathbb{R}$.

$$A = \begin{pmatrix} 1 & 3 \\ 1 & -1 \end{pmatrix} .$$

V1.3. Linear algebra. List of problems.

V1.3.1. Solve the following homogeneous system of linear equations. Determine a fundamental system of solutions.

$$\begin{cases} 5x_1 - 2x_2 + 3x_3 - 7x_4 + 4x_5 = 0 \\ x_1 - 2x_2 + x_3 + 3x_4 - 2x_5 = 0 \\ 2x_1 + 3x_2 + 7x_3 - 2x_4 + 3x_5 = 0 \end{cases}$$

V1.3.2. Determine the general solution of the following non-homogeneous system of linear equations.

$$\begin{cases} 7x_1 + 2x_2 - x_3 - 15x_4 = 6 \\ x_1 - 3x_2 + x_3 = -10 \\ 2x_1 - 2x_2 + x_3 - 3x_4 = -9 \\ x_1 + x_2 + x_3 - 2x_4 = 2 \end{cases}$$

V1.3.3. Determine the characteristic polynomial, the eigenvalues and the eigenvectors of the following matrix A. Write down the corresponding diagonal matrix D and a transition matrix C.

$$A = \begin{pmatrix} -7 & 6 & 0 \\ -21 & 15 & 1 \\ -9 & 6 & 2 \end{pmatrix} .$$

V1.3.4. Factor the characteristic polynomial over $\mathbb{C}$ and determine the Jordan form of the following matrix A.

$$A = \begin{pmatrix} -20 & 51 & 17 & 29 \\ 84 & -176 & -54 & -99 \\ 125 & -278 & -86 & -156 \\ -230 & 506 & 158 & 285 \end{pmatrix} .$$

V1.3.5. Determine an orthogonal basis of a subspace $L \subset \mathbb{R}^4$ spanned by the following vectors

$$V_1 = (1,1,1,1),\ V_2 = (-3,1,-2,0),\ V_3 = (2,4,5,1) .$$

V1.3.6. Determine the distance between the following vector X (the endpoint of X) and a subspace $L \subset \mathbb{R}^4$ given as the subspace of solutions of the following homogeneous linear system:

$$X = (9,-1,-3,1) , \quad L : \begin{cases} x_2 + 7x_3 - 4x_4 = 0 \\ x_1 + 2x_2 + 2x_3 - x_4 = 0 . \end{cases}$$

V1.3.7. Determine $\exp(tA)$ where A is a given matrix, $t \in \mathbb{R}$.

$$A = \begin{pmatrix} 1 & 4 \\ 2 & -1 \end{pmatrix} .$$

V1.4. Linear algebra. List of problems.

V1.4.1. Solve the following homogeneous system of linear equations. Determine a fundamental system of solutions.

$$\begin{cases} 2x_1 - 2x_2 - 3x_3 + 3x_4 + \ \ x_5 = 0 \\ \ \ x_1 - 3x_2 + \ \ x_3 + 3x_4 + 9x_5 = 0 \\ 4x_1 - 5x_2 - 5x_3 + 7x_4 + 6x_5 = 0 \end{cases}$$

V1.4.2. Determine the general solution of the following non-homogeneous system of linear equations.

$$\begin{cases} 3x_1 + 2x_2 - 5x_3 - 15x_4 & = & -15 \\ \ \ x_1 + 3x_2 + \ \ x_3 - \ \ 5x_4 & = & 10 \\ 5x_1 - 4x_2 + 5x_3 - \ \ 6x_4 & = & -7 \\ 6x_1 - 3x_2 - 2x_3 + \ \ 4x_4 & = & -7 \end{cases}$$

V1.4.3. Determine the characteristic polynomial, the eigenvalues and the eigenvectors of the following matrix A. Write down the corresponding diagonal matrix D and a transition matrix C.

$$A = \begin{pmatrix} -24 & 5 & -7 \\ 42 & -8 & 12 \\ 108 & -21 & 31 \end{pmatrix} .$$

V1.4.4. Factor the characteristic polynomial over $\mathbb{C}$ and determine the Jordan form of the following matrix A.

$$A = \begin{pmatrix} 96 & -69 & -22 & -46 \\ -327 & 233 & 75 & 156 \\ -528 & 376 & 116 & 249 \\ 955 & -682 & -217 & -455 \end{pmatrix} .$$

V1.4.5. Determine an orthogonal basis of a subspace $L \subset \mathbb{R}^4$ spanned by the following vectors

$$V_1 = (-1, 1, -1, 1),\ V_2 = (-1, -3, 0, -2),\ V_3 = (1, 5, -3, 5)\ .$$

V1.4.6. Determine the distance between the following vector X (the endpoint of X) and a subspace $L \subset \mathbb{R}^4$ given as the subspace of solutions of the following homogeneous linear system:

$$X = (1, 3, 6, 0)\ , \quad L: \begin{cases} -2x_1 + \ \ x_2 + \ \ x_3 - x_4 = 0 \\ -\ x_1 + 4x_2 + 7x_3 - x_4 = 0\,. \end{cases}$$

V1.4.7. Determine $\exp(tA)$ where A is a given matrix, $t \in \mathbb{R}$.

$$A = \begin{pmatrix} 1 & 2 \\ 4 & -1 \end{pmatrix} .$$

V1.5. Linear algebra. List of problems.

V1.5.1. Solve the following homogeneous system of linear equations. Determine a fundamental system of solutions.

$$\begin{cases} 3x_1 - x_2 + 8x_3 - 11x_4 + 3x_5 = 0 \\ x_1 - 2x_2 - 5x_3 - 4x_4 = 0 \\ 7x_1 - 4x_2 + 3x_3 + x_4 - 18x_5 = 0 \end{cases}$$

V1.5.2. Determine the general solution of the following non-homogeneous system of linear equations.

$$\begin{cases} 5x_1 - 2x_2 - 3x_3 - 11x_4 = -7 \\ x_1 - 2x_2 + x_3 - 7x_4 = -3 \\ 2x_1 - 3x_2 - 7x_3 - 3x_4 = -21 \\ x_1 - x_2 + 3x_3 + x_4 = 4 \end{cases}$$

V1.5.3. Determine the characteristic polynomial, the eigenvalues and the eigenvectors of the following matrix A. Write down the corresponding diagonal matrix D and a transition matrix C.

$$A = \begin{pmatrix} 85 & 19 & 11 \\ -150 & -29 & -21 \\ -408 & -85 & -55 \end{pmatrix} .$$

V1.5.4. Factor the characteristic polynomial over $\mathbb{C}$ and determine the Jordan form of the following matrix A.

$$A = \begin{pmatrix} -11 & 18 & 43 & 28 \\ -37 & 52 & 111 & 75 \\ -12 & 15 & 28 & 20 \\ 36 & -47 & -94 & -65 \end{pmatrix} .$$

V1.5.5. Determine an orthogonal basis of a subspace $L \subset \mathbb{R}^4$ spanned by the following vectors

$$V_1 = (1, 2, 2, 0),\ V_2 = (2, 5, 3, -1),\ V_3 = (-1, -2, -2,\ 21) .$$

V1.5.6. Determine the distance between the following vector X (the endpoint of X) and a subspace $L \subset \mathbb{R}^4$ given as the subspace of solutions of the following homogeneous linear system:

$$X = (5, 4, -2, -1) , \quad L : \begin{cases} 2x_1 + 3x_2 - 3x_3 + 2x_4 = 0 \\ x_1 + 2x_2 + 2x_3 - x_4 = 0 . \end{cases}$$

V1.5.7. Determine $\exp(tA)$ where A is a given matrix, $t \in \mathbb{R}$.

$$A = \begin{pmatrix} 2 & 5 \\ 1 & -2 \end{pmatrix} .$$

V1.6. Linear algebra. List of problems.

V1.6.1. Solve the following homogeneous system of linear equations. Determine a fundamental system of solutions.

$$\begin{cases} 3x_1 + x_2 - 8x_3 + 2x_4 - 18x_5 = 0 \\ x_1 - 2x_2 + 3x_3 + 2x_4 + 3x_5 = 0 \\ x_1 + 6x_2 - 9x_3 - 10x_4 - 13x_5 = 0 \end{cases}$$

V1.6.2. Determine the general solution of the following non-homogeneous system of linear equations.

$$\begin{cases} 4x_1 + x_2 - x_3 & = & -8 \\ x_1 + 2x_2 - x_3 + 2x_4 & = & -1 \\ 2x_1 - 3x_2 + x_3 - 4x_4 & = & -10 \\ 8x_1 + 2x_2 - 2x_3 & = & 10 \end{cases}$$

V1.6.3. Determine the characteristic polynomial, the eigenvalues and the eigenvectors of the following matrix A. Write down the corresponding diagonal matrix D and a transition matrix C.

$$A = \begin{pmatrix} 8 & -2 & 2 \\ 15 & -3 & 5 \\ -6 & 2 & 0 \end{pmatrix} .$$

V1.6.4. Factor the characteristic polynomial over $\mathbb{C}$ and determine the Jordan form of the following matrix A.

$$A = \begin{pmatrix} 48 & -42 & 29 & -19 \\ 139 & -120 & 81 & -54 \\ 40 & -34 & 23 & -15 \\ -130 & 111 & -76 & 49 \end{pmatrix} .$$

V1.6.5. Determine an orthogonal basis of a subspace $L \subset \mathbb{R}^4$ spanned by the following vectors

$$V_1 = (3, -1, 0, 0),\ V_2 = (-2, 4, 1, 2),\ V_3 = (1, 3, 5, 5)\ .$$

V1.6.6. Determine the distance between the following vector X (the endpoint of X) and a subspace $L \subset \mathbb{R}^4$ given as the subspace of solutions of the following homogeneous linear system:

$$X = (-3, 2, 6, 14)\ , \quad L : \begin{cases} -5x_1 + 6x_2 + 9x_3 - 3x_4 = 0 \\ -3x_1 + 5x_2 + 8x_3 - 2x_4 = 0\,. \end{cases}$$

V1.6.7. Determine $\exp(tA)$ where A is a given matrix, $t \in \mathbb{R}$.

$$A = \begin{pmatrix} -2 & 5 \\ 1 & 2 \end{pmatrix} .$$

V1.7. Linear algebra. List of problems.

V1.7.1. Solve the following homogeneous system of linear equations. Determine a fundamental system of solutions.

$$\begin{cases} 2x_1 + x_2 + x_3 - 5x_4 - x_5 = 0 \\ x_1 + 4x_2 - 2x_3 - 3x_4 - 12x_5 = 0 \\ x_1 + x_2 + x_3 - 6x_4 + 3x_5 = 0 \end{cases}$$

V1.7.2. Determine the general solution of the following non-homogeneous system of linear equations.

$$\begin{cases} 2x_1 - 2x_2 - 3x_3 - 7x_4 &= -15 \\ x_1 - 3x_2 + x_3 + 2x_4 &= -7 \\ 4x_1 + 5x_2 + 5x_3 - 7x_4 &= -10 \\ 2x_1 - x_2 + x_3 + x_4 &= 3 \end{cases}$$

V1.7.3. Determine the characteristic polynomial, the eigenvalues and the eigenvectors of the following matrix A. Write down the corresponding diagonal matrix D and a transition matrix C.

$$A = \begin{pmatrix} 3 & 8 & 4 \\ 6 & 21 & 10 \\ -12 & -44 & -21 \end{pmatrix} .$$

V1.7.4. Factor the characteristic polynomial over $\mathbb{C}$ and determine the Jordan form of the following matrix A.

$$A = \begin{pmatrix} -35 & 27 & 7 & 21 \\ -68 & 51 & 11 & 38 \\ 5 & -4 & 0 & -3 \\ 26 & -18 & -2 & -12 \end{pmatrix} .$$

V1.7.5. Determine an orthogonal basis of a subspace $L \subset \mathbb{R}^4$ spanned by the following vectors

$$V_1 = (2, 1, 3, -1),\ V_2 = (5, 0, 1, -2),\ V_3 = (-10, 0, 1, -17) .$$

V1.7.6. Determine the distance between the following vector X (the endpoint of X) and a subspace $L \subset \mathbb{R}^4$ given as the subspace of solutions of the following homogeneous linear system:

$$X = (1, 1, 7, -1) , \quad L : \begin{cases} 3x_1 - 5x_2 - 8x_3 + 2x_4 = 0 \\ 4x_1 - 9x_2 - 15x_3 + 3x_4 = 0 . \end{cases}$$

V1.7.7. Determine $\exp(tA)$ where A is a given matrix, $t \in \mathbb{R}$.

$$A = \begin{pmatrix} 3 & 1 \\ -5 & -3 \end{pmatrix} .$$

V1.8. Linear algebra. List of problems.

V1.8.1. Solve the following homogeneous system of linear equations. Determine a fundamental system of solutions.

$$\begin{cases} x_1 + x_2 - x_3 - 5x_4 + 5x_5 = 0 \\ 3x_1 + x_2 - 2x_3 - 8x_4 - x_5 = 0 \\ 2x_1 - 2x_2 + x_3 + 5x_4 - 8x_5 = 0 \end{cases}$$

V1.8.2. Determine the general solution of the following non-homogeneous system of linear equations.

$$\begin{cases} 2x_1 - 2x_2 - 3x_3 + 10x_4 = 7 \\ x_1 - 3x_2 + x_3 + 8x_4 = 3 \\ 3x_1 - x_2 - 3x_3 + 6x_4 = 7 \\ 3x_1 + 2x_2 - 2x_3 - x_4 = 11 \end{cases}$$

V1.8.3. Determine the characteristic polynomial, the eigenvalues and the eigenvectors of the following matrix A. Write down the corresponding diagonal matrix D and a transition matrix C.

$$A = \begin{pmatrix} -3 & 2 & 0 \\ -13 & 5 & -1 \\ 31 & -8 & 4 \end{pmatrix} .$$

V1.8.4. Factor the characteristic polynomial over $\mathbb{C}$ and determine the Jordan form of the following matrix A.

$$A = \begin{pmatrix} -6 & 3 & 4 & 3 \\ 0 & -2 & 5 & 2 \\ 5 & -4 & -5 & -3 \\ -8 & 6 & 1 & 1 \end{pmatrix} .$$

V1.8.5. Determine an orthogonal basis of a subspace $L \subset \mathbb{R}^4$ spanned by the following vectors

$$V_1 = (-3, 2, -1, 1),\ V_2 = (5, -4, 4, -3),\ V_3 = (1, -2, 5, -3) .$$

V1.8.6. Determine the distance between the following vector X (the endpoint of X) and a subspace $L \subset \mathbb{R}^4$ given as the subspace of solutions of the following homogeneous linear system:

$$X = (-3, 6, -4, 1) , \quad L : \begin{cases} 3x_1 + 5x_2 - x_3 + x_4 = 0 \\ x_2 + 7x_3 - 4x_4 = 0 . \end{cases}$$

V1.8.7. Determine $\exp(tA)$ where A is a given matrix, $t \in \mathbb{R}$.

$$A = \begin{pmatrix} -3 & 4 \\ -2 & 3 \end{pmatrix} .$$

V1.9. Linear algebra. List of problems.

V1.9.1. Solve the following homogeneous system of linear equations. Determine a fundamental system of solutions.

$$\begin{cases} x_1 + 2x_2 + 3x_3 - x_4 - 2x_5 = 0 \\ 2x_1 - x_2 + x_3 - 7x_4 + 6x_5 = 0 \\ 4x_1 + 7x_2 + 5x_3 + x_4 - 12x_5 = 0 \end{cases}$$

V1.9.2. Determine the general solution of the following non-homogeneous system of linear equations.

$$\begin{cases} x_1 + 2x_2 + 3x_3 - 4x_4 = 4 \\ 2x_1 - x_2 + x_3 + 2x_4 = 8 \\ 4x_1 + 7x_2 + 5x_3 - 20x_4 = 4 \\ 3x_1 + 2x_2 - 2x_3 - x_4 = 8 \end{cases}$$

V1.9.3. Determine the characteristic polynomial, the eigenvalues and the eigenvectors of the following matrix A. Write down the corresponding diagonal matrix D and a transition matrix C.

$$A = \begin{pmatrix} 3 & -2 & -1 \\ 19 & -11 & -5 \\ -31 & 16 & 7 \end{pmatrix} .$$

V1.9.4. Factor the characteristic polynomial over $\mathbb{C}$ and determine the Jordan form of the following matrix A.

$$A = \begin{pmatrix} -27 & 18 & 16 & 7 \\ -79 & 52 & 45 & 21 \\ 24 & -16 & -13 & -8 \\ 27 & -17 & -15 & -5 \end{pmatrix} .$$

V1.9.5. Determine an orthogonal basis of a subspace $L \subset \mathbb{R}^4$ spanned by the following vectors

$$V_1 = (1, 2, 4, 2),\ V_2 = (-1, 3, 5, 0),\ V_3 = (5, -5, -3, -4) .$$

V1.9.6. Determine the distance between the following vector X (the endpoint of X) and a subspace $L \subset \mathbb{R}^4$ given as the subspace of solutions of the following homogeneous linear system:

$$X = (-1, 3, 2, 4) , \quad L : \begin{cases} x_1 + x_2 + x_3 + x_4 = 0 \\ -3x_1 + x_2 - 2x_3 = 0 . \end{cases}$$

V1.9.7. Determine $\exp(tA)$ where A is a given matrix, $t \in \mathbb{R}$.

$$A = \begin{pmatrix} -2 & 6 \\ 2 & 0 \end{pmatrix} .$$

V1.10. Linear algebra. List of problems.

V1.10.1. Solve the following homogeneous system of linear equations. Determine a fundamental system of solutions.

$$\begin{cases} 7x_1 + 2x_2 - 5x_3 - 6x_4 - 3x_5 = 0 \\ x_1 + 3x_2 + x_3 - 8x_4 + 6x_5 = 0 \\ 5x_1 - 4x_2 - 5x_3 + 8x_4 - 17x_5 = 0 \end{cases}$$

V1.10.2. Determine the general solution of the following non-homogeneous system of linear equations.

$$\begin{cases} 3x_1 + 2x_2 - 3x_3 + 7x_4 &= 3 \\ x_1 - 2x_2 - 4x_3 + 3x_4 &= -4 \\ x_1 + x_2 - x_3 + 3x_4 &= 2 \\ 5x_1 - 3x_2 - 5x_3 - 3x_4 &= -7 \end{cases}$$

V1.10.3. Determine the characteristic polynomial, the eigenvalues and the eigenvectors of the following matrix A. Write down the corresponding diagonal matrix D and a transition matrix C.

$$A = \begin{pmatrix} 10 & 3 & 3 \\ -45 & -14 & -15 \\ 27 & 9 & 10 \end{pmatrix} .$$

V1.10.4. Factor the characteristic polynomial over $\mathbb{C}$ and determine the Jordan form of the following matrix A.

$$A = \begin{pmatrix} -14 & 7 & 22 & 4 \\ -13 & 6 & 28 & 4 \\ -22 & 12 & 20 & 5 \\ 20 & -10 & -25 & -4 \end{pmatrix} .$$

V1.10.5. Determine an orthogonal basis of a subspace $L \subset \mathbb{R}^4$ spanned by the following vectors

$$V_1 = (4, 1, 2, -2),\ V_2 = (5, -1, 3, 0),\ V_3 = (8, 11, 2, -14) .$$

V1.10.6. Determine the distance between the following vector X (the endpoint of X) and a subspace $L \subset \mathbb{R}^4$ given as the subspace of solutions of the following homogeneous linear system:

$$X = (-2, 2, 1, 3) , \quad L : \begin{cases} -2x_1 + 2x_2 - x_3 + x_4 = 0 \\ x_1 + x_2 + x_3 + x_4 = 0 . \end{cases}$$

V1.10.7. Determine $\exp(tA)$ where A is a given matrix, $t \in \mathbb{R}$.

$$A = \begin{pmatrix} 2 & -2 \\ -4 & 0 \end{pmatrix} .$$

V1.11. Linear algebra. List of problems.

V1.11.1. Solve the following homogeneous system of linear equations. Determine a fundamental system of solutions.

$$\begin{cases} 4x_1 + x_2 - 7x_3 + x_4 + 8x_5 = 0 \\ x_1 + 2x_2 - x_3 - 3x_4 - 4x_5 = 0 \\ 2x_1 - 3x_2 + 7x_3 - 17x_4 + 4x_5 = 0 \end{cases}$$

V1.11.2. Determine the general solution of the following non-homogeneous system of linear equations.

$$\begin{cases} x_1 - 3x_2 + 2x_3 - 4x_4 &= 9 \\ 2x_1 + x_2 - 3x_3 - x_4 &= -10 \\ 3x_1 - 2x_2 - x_3 - 5x_4 &= -1 \\ 2x_1 - 3x_2 - x_3 + x_4 &= 4 \end{cases}$$

V1.11.3. Determine the characteristic polynomial, the eigenvalues and the eigenvectors of the following matrix A. Write down the corresponding diagonal matrix D and a transition matrix C.

$$A = \begin{pmatrix} -6 & -3 & -1 \\ 1 & 2 & -1 \\ 23 & 9 & 6 \end{pmatrix} .$$

V1.11.4. Factor the characteristic polynomial over $\mathbb{C}$ and determine the Jordan form of the following matrix A.

$$A = \begin{pmatrix} 25 & -19 & -18 & -13 \\ 72 & -53 & -49 & -35 \\ -19 & 13 & 11 & 8 \\ -27 & 20 & 19 & 13 \end{pmatrix} .$$

V1.11.5. Determine an orthogonal basis of a subspace $L \subset \mathbb{R}^4$ spanned by the following vectors

$$V_1 = (0, 3, 1, 0),\ V_2 = (5, 1, -3, 5),\ V_3 = (8, -2, -4, 2) .$$

V1.11.6. Determine the distance between the following vector X (the endpoint of X) and a subspace $L \subset \mathbb{R}^4$ given as the subspace of solutions of the following homogeneous linear system:

$$X = (-1, 5, 3, 1) , \quad L : \begin{cases} 4x_1 + 3x_3 + x_4 = 0 \\ -x_1 + 3x_2 + 2x_4 = 0 . \end{cases}$$

V1.11.7. Determine $\exp(tA)$ where A is a given matrix, $t \in \mathbb{R}$.

$$A = \begin{pmatrix} 2 & 3 \\ 4 & -2 \end{pmatrix} .$$

V1.12. Linear algebra. List of problems.

V1.12.1. Solve the following homogeneous system of linear equations. Determine a fundamental system of solutions.

$$\begin{cases} 2x_1 - 2x_2 - 3x_3 + 4x_4 + 10\,x_5 = 0 \\ x_1 - 3x_2 + x_3 + x_4 + 16\,x_5 = 0 \\ 3x_1 - x_2 + 3x_3 - x_4 + 12\,x_5 = 0 \end{cases}$$

V1.12.2. Determine the general solution of the following non-homogeneous system of linear equations.

$$\begin{cases} x_1 - 3x_2 + 4x_3 - x_4 = 1 \\ 2x_1 + x_2 - 2x_3 - 3x_4 = 8 \\ 2x_1 - 2x_2 + 3x_3 - 4x_4 = 4 \\ 2x_1 + x_2 - 2x_3 - 6x_4 = 5 \end{cases}$$

V1.12.3. Determine the characteristic polynomial, the eigenvalues and the eigenvectors of the following matrix A. Write down the corresponding diagonal matrix D and a transition matrix C.

$$A = \begin{pmatrix} 6 & 5 & 7 \\ 4 & 16 & 16 \\ -4 & -11 & -11 \end{pmatrix}.$$

V1.12.4. Factor the characteristic polynomial over $\mathbb{C}$ and determine the Jordan form of the following matrix A.

$$A = \begin{pmatrix} 23 & -18 & 7 & -12 \\ 70 & -52 & 19 & -33 \\ -18 & 12 & -5 & 7 \\ -65 & 46 & -17 & 28 \end{pmatrix}.$$

V1.12.5. Determine an orthogonal basis of a subspace $L \subset \mathbb{R}^4$ spanned by the following vectors

$$V_1 = (-2, 2, -4, 1),\ V_2 = (0, 3, -5, -1),\ V_3 = (4, -5, 3, 5)\,.$$

V1.12.6. Determine the distance between the following vector X (the endpoint of X) and a subspace $L \subset \mathbb{R}^4$ given as the subspace of solutions of the following homogeneous linear system:

$$X = (1, -5, -9, -3)\,, \quad L: \begin{cases} -x_1 + x_2 - x_3 + x_4 = 0 \\ -x_1 - 3x_2 - 2x_4 = 0\,. \end{cases}$$

V1.12.7. Determine $\exp(tA)$ where A is a given matrix, $t \in \mathbb{R}$.

$$A = \begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix}.$$