V4.1. Calculus of variations. List of problems.

V4.1.1. Find the family of extremals for the following classical variational problem (find the general solution of the Euler-Lagrange equation)

$$\mathcal{J}[x(t)] = \int_{t_0}^{t_1} L(t, x(t), \dot{x}(t))\, dt \ \to \text{extr}\ ,$$

$$L = 1 + t^2 + 2tx + x^2 - x\dot{x} + \dot{x}^2 + 2t\dot{x}\ .$$

V4.1.2. Find all extremals for the following classical variational problem

$$\mathcal{J}[x(t)] = \int_0^2 (t + \dot{x}^3)\, dt \ \to \text{extr}\ , \quad x(0) = 0\,,\ x(2) = 4\ .$$

V4.1.3. Find the extremals for the Bolza problem

$$\mathcal{B}[x(t)] = \int_1^2 t^2\dot{x}^2\, dt - 2x(1) + x^2(2) \ \to \text{extr}\ \ .$$

V4.1.4. Find the extremals for the isoperimetric problem.

$$\mathcal{J}_0[x(t)] = \int_0^1 \dot{x}^2\, dt \ \to \text{extr}\ , \quad \int_0^1 x\, dt = 1\ ,\ x(0) = 0\ ,\ x(1) = 0.$$

V4.2. Calculus of variations. List of problems.

V4.2.1. Find the family of extremals for the following classical variational problem (find the general solution of the Euler-Lagrange equation)

$$\mathcal{J}[x(t)] = \int_{t_0}^{t_1} L(t, x(t), \dot{x}(t))\, dt \ \to \text{extr}\ ,$$

$$L = t - t^3 + 2tx - x^2 + 3x\dot{x} - \dot{x}^2 - 4t\dot{x}\ .$$

V4.2.2. Find all extremals for the following classical variational problem

$$\mathcal{J}[x(t)] = \int_0^1 (2\dot{x}^3 + \dot{x} + 1)\, dt \ \to \text{extr}\ , \quad x(0) = 0\,,\ x(1) = 3\ .$$

V4.2.3. Find the extremals for the Bolza problem

$$\mathcal{B}[x(t)] = \int_1^e (2t\dot{x}^2 + 2x\dot{x})\, dt + 3x^2(1) - x^2(e) - 4x(e) \ \to \text{extr}\ \ .$$

V4.2.4. Find the extremals for the isoperimetric problem.

$$\mathcal{J}_0[x(t)] = \int_0^\pi \dot{x}^2\, dt \ \to \text{extr}\ , \quad \int_0^1 x \sin t\, dt = 1\ ,\ x(0) = 0\ ,\ x(\pi) = 0.$$

V4.3. Calculus of variations. List of problems.

V4.3.1. Find the family of extremals for the following classical variational problem (find the general solution of the Euler-Lagrange equation)

$$\mathcal{J}[x(t)] = \int_{t_0}^{t_1} L(t, x(t), \dot{x}(t))\, dt \ \to \text{extr}\ ,$$

$$L = t^4 + tx + 2x^2 - 5x\dot{x} + 2\dot{x}^2 - t\dot{x}\ .$$

V4.3.2. Find all extremals for the following classical variational problem

$$\mathcal{J}[x(t)] = \int_0^3 (t^3 - \dot{x}^3)\, dt \ \to \text{extr}\ , \quad x(0) = 0\,,\ x(3) = 3\ .$$

V4.3.3. Find the extremals for the Bolza problem

$$\mathcal{B}[x(t)] = \int_0^1 e^x \dot{x}^2\, dt + 4e^{x(0)} + 32e^{-x(1)} \ \to \text{extr}\ \ .$$

V4.3.4. Find the extremals for the isoperimetric problem.

$$\mathcal{J}_0[x(t)] = \int_1^2 t^2\, \dot{x}^2\, dt \ \to \text{extr}\ , \quad \int_1^2 tx\, dt = 7/3\ ,\ x(1) = 1\ ,\ x(2) = 2.$$

V4.4. Calculus of variations. List of problems.

V4.4.1. Find the family of extremals for the following classical variational problem (find the general solution of the Euler-Lagrange equation)

$$\mathcal{J}[x(t)] = \int_{t_0}^{t_1} L(t, x(t), \dot{x}(t))\, dt \ \to \text{extr}\ ,$$

$$L = 1 + e^{2t} + 3\dot{x} - 2tx + x^2 + 6x\dot{x} + 4\dot{x}^2\ .$$

V4.4.2. Find all extremals for the following classical variational problem

$$\mathcal{J}[x(t)] = \int_0^2 (t^2 + \dot{x} + \dot{x}^3)\, dt \ \to \text{extr}\ , \quad x(0) = 0\,,\ x(2) = 3\ .$$

V4.4.3. Find the extremals for the Bolza problem

$$\mathcal{B}[x(t)] = \int_0^1 e^{t+1}(\dot{x}^2 + 2x^2)\, dt + 2x(0)x(1) + 2x(1) \ \to \text{extr}\ \ .$$

V4.4.4. Find the extremals for the isoperimetric problem.

$$\mathcal{J}_0[x(t)] = \int_0^1 (\dot{x}^2 + x^2)\, dt \ \to \text{extr}\ , \quad \int_0^1 xe^t\, dt = \frac{e^2+1}{4}\ ,\ x(0) = 0\ ,\ x(1) = e.$$

V4.5. Calculus of variations. List of problems.

V4.5.1. Find the family of extremals for the following classical variational problem (find the general solution of the Euler-Lagrange equation)

$$\mathcal{J}[x(t)] = \int_{t_0}^{t_1} L(t, x(t), \dot{x}(t))\, dt \ \to \text{extr}\ ,$$

$$L = 1 + t^2 + 2tx - 2x^2 - 11x\dot{x} - 8\dot{x}^2 + (t+1)\dot{x}\ .$$

V4.5.2. Find all extremals for the following classical variational problem

$$\mathcal{J}[x(t)] = \int_0^2 (4t\dot{x} - \dot{x}^2)\, dt \ \to \text{extr}\ , \quad x(0) = 0\,,\ x(2) = 4\ .$$

V4.5.3. Find the extremals for the Bolza problem

$$\mathcal{B}[x(t)] = \int_0^{\pi/2} (\dot{x}^2 - x^2 - 2x)\, dt - 2x^2(0) - x^2(\pi/2) \ \to \text{extr}\ \ .$$

V4.5.4. Find the extremals for the isoperimetric problem.

$$\mathcal{J}_0[x(t)] = \int_0^{\pi} (\dot{x}^2 - x^2)\, dt \ \to \text{extr}\ , \quad \int_0^{\pi} x\cos t\, dt = 1\ ,\ x(0) = 0\ ,\ x(\pi) = 0.$$

V4.6. Calculus of variations. List of problems.

V4.6.1. Find the family of extremals for the following classical variational problem (find the general solution of the Euler-Lagrange equation)

$$\mathcal{J}[x(t)] = \int_{t_0}^{t_1} L(t, x(t), \dot{x}(t))\, dt \ \to \text{extr}\ ,$$

$$L = t^3 + t^4 + 6tx + x^2 - 2x\dot{x} + 4\dot{x}^2\ .$$

V4.6.2. Find all extremals for the following classical variational problem

$$\mathcal{J}[x(t)] = \int_0^1 (t^2 + 2\dot{x}^2 - 8t\dot{x})\, dt \ \to \text{extr}\ , \quad x(0) = 0\,,\ x(1) = 1\ .$$

V4.6.3. Find the extremals for the Bolza problem

$$\mathcal{B}[x(t)] = \int_0^{e-1} (t+1)\dot{x}^2\, dt + 2x(0) + 2x(0)\, x(e-1) \ \to \text{extr}\ \ .$$

V4.6.4. Find the extremals for the isoperimetric problem.

$$\mathcal{J}_0[x(t)] = \int_1^2 t\, \dot{x}^2\, dt \ \to \text{extr}\ , \quad \int_1^2 tx\, dt = 21/4\ ,\ x(1) = 2\ ,\ x(2) = 5\,.$$

V4.7. Calculus of variations. List of problems.

V4.7.1. Find the family of extremals for the following classical variational problem (find the general solution of the Euler-Lagrange equation)

$$\mathcal{J}[x(t)] = \int_{t_0}^{t_1} L(t, x(t), \dot{x}(t))\, dt \ \to \text{extr} \ ,$$

$$L = 1 + \cos^2 t - tx - x^2 + 3x\dot{x} - 4\dot{x}^2 \ .$$

V4.7.2. Find all extremals for the following classical variational problem

$$\mathcal{J}[x(t)] = \int_0^3 \left(t^3 + 2t\dot{x} - \frac{1}{2}\,\dot{x}^2 \right) dt \ \to \text{extr} \ , \quad x(0) = 0 \, , \ x(3) = 0 \ .$$

V4.7.3. Find the extremals for the Bolza problem

$$\mathcal{B}[x(t)] = \int_1^2 t^2\dot{x}^2\, dt - 2x(1) + x^2(2) \ \to \text{extr} \ \ .$$

V4.7.4. Find the extremals for the isoperimetric problem.

$$\mathcal{J}_0[x(t)] = \int_0^1 \dot{x}^2\, dt \ \to \text{extr} \ , \quad \int_0^1 x\, dt = 1 \ , \ x(0) = 0 \ , \ x(1) = 0.$$

V4.8. Calculus of variations. List of problems.

V4.8.1. Find the family of extremals for the following classical variational problem (find the general solution of the Euler-Lagrange equation)

$$\mathcal{J}[x(t)] = \int_{t_0}^{t_1} L(t, x(t), \dot{x}(t))\, dt \ \to \text{extr} \ ,$$

$$L = \sin 4t - 7tx - 2x^2 + x\dot{x} - 8\dot{x}^2 + \dot{x} \ .$$

V4.8.2. Find all extremals for the following classical variational problem

$$\mathcal{J}[x(t)] = \int_0^1 (t^2 + \dot{x}^2 - 4t\dot{x})\, dt \ \to \text{extr} \ , \quad x(0) = 0 \, , \ x(1) = 0 \ .$$

V4.8.3. Find the extremals for the Bolza problem

$$\mathcal{B}[x(t)] = \int_1^e (2t\dot{x}^2 + 2x\dot{x})\, dt + 3x^2(1) - x^2(e) - 4x(e) \ \to \text{extr} \ \ .$$

V4.8.4. Find the extremals for the isoperimetric problem.

$$\mathcal{J}_0[x(t)] = \int_0^\pi \dot{x}^2\, dt \ \to \text{extr} \ , \quad \int_0^\pi x \sin t\, dt = 1 \ , \ x(0) = 0 \ , \ x(\pi) = 0.$$

V4.9. Calculus of variations. List of problems.

V4.9.1. Find the family of extremals for the following classical variational problem (find the general solution of the Euler-Lagrange equation)

$$\mathcal{J}[x(t)] = \int_{t_0}^{t_1} L(t, x(t), \dot{x}(t))\, dt \ \to \text{extr}\ ,$$

$$L = e^t + e^{-t} + tx + 4x^2 - 12x\dot{x} + \dot{x}^2 - 3t\dot{x}\ .$$

V4.9.2. Find all extremals for the following classical variational problem

$$\mathcal{J}[x(t)] = \int_0^{\pi/2} (t^2 - x^2 + \dot{x}^2)\, dt \ \to \text{extr}\ , \quad x(0) = 0\,,\ x(\pi/2) = 1\ .$$

V4.9.3. Find the extremals for the Bolza problem

$$\mathcal{B}[x(t)] = \int_0^1 e^x \dot{x}^2\, dt + 4e^{x(0)} + 32e^{-x(1)} \ \to \text{extr}\ \ .$$

V4.9.4. Find the extremals for the isoperimetric problem.

$$\mathcal{J}_0[x(t)] = \int_1^2 t^2\, \dot{x}^2\, dt \ \to \text{extr}\ , \quad \int_1^2 tx\, dt = 7/3\ ,\ x(1) = 1\ ,\ x(2) = 2.$$

V4.10. Calculus of variations. List of problems.

V4.10.1. Find the family of extremals for the following classical variational problem (find the general solution of the Euler-Lagrange equation)

$$\mathcal{J}[x(t)] = \int_{t_0}^{t_1} L(t, x(t), \dot{x}(t))\, dt \ \to \text{extr}\ ,$$

$$L = 4 + t^4 - 4tx - 8x^2 + x\dot{x} - 2\dot{x}^2 - 3\dot{x} + 5t\dot{x}\ .$$

V4.10.2. Find all extremals for the following classical variational problem

$$\mathcal{J}[x(t)] = \int_0^{3\pi/2} (1 - x^2 + \dot{x}^2)\, dt \ \to \text{extr}\ , \quad x(0) = 0\,,\ x(3\pi/2) = 1\ .$$

V4.10.3. Find the extremals for the Bolza problem

$$\mathcal{B}[x(t)] = \int_0^1 e^{t+1}(\dot{x}^2 + 2x^2)\, dt + 2x(0)x(1) + 2x(1) \ \to \text{extr}\ \ .$$

V4.10.4. Find the extremals for the isoperimetric problem.

$$\mathcal{J}_0[x(t)] = \int_0^1 (\dot{x}^2 + x^2)\, dt \ \to \text{extr}\ , \quad \int_0^1 xe^t\, dt = \frac{e^2+1}{4}\ ,\ x(0) = 0\ ,\ x(1) = e.$$

V4.11. Calculus of variations. List of problems.

V4.11.1. Find the family of extremals for the following classical variational problem (find the general solution of the Euler-Lagrange equation)

$$\mathcal{J}[x(t)] = \int_{t_0}^{t_1} L(t, x(t), \dot{x}(t))\, dt \ \to \text{extr}\ ,$$

$$L = \sin^2 \frac{t}{2} + tx + x^2 + 6x\dot{x} + \frac{1}{4}\,\dot{x}^2 + t\dot{x}\ .$$

V4.11.2. Find all extremals for the following classical variational problem

$$\mathcal{J}[x(t)] = \int_0^1 (x^2 - \dot{x}^2 + 2\dot{x})\, dt \ \to \text{extr}\ , \quad x(0) = 0\,,\ x(1) = \cos 1\ .$$

V4.11.3. Find the extremals for the Bolza problem

$$\mathcal{B}[x(t)] = \int_0^{\pi/2} (\dot{x}^2 - x^2 - 2x)\, dt - 2x^2(0) - x^2(\pi/2) \ \to \text{extr}\ \ .$$

V4.11.4. Find the extremals for the isoperimetric problem.

$$\mathcal{J}_0[x(t)] = \int_0^{\pi} (\dot{x}^2 - x^2)\, dt \ \to \text{extr}\ , \quad \int_0^{\pi} x \cos t\, dt = 1\ ,\ x(0) = 0\ ,\ x(\pi) = 0.$$

V4.12. Calculus of variations. List of problems.

V4.12.1. Find the family of extremals for the following classical variational problem (find the general solution of the Euler-Lagrange equation)

$$\mathcal{J}[x(t)] = \int_{t_0}^{t_1} L(t, x(t), \dot{x}(t))\, dt \ \to \text{extr}\ ,$$

$$L = 3 + t^3 - tx - 2x^2 + 5x\dot{x} - \frac{1}{2}\,\dot{x}^2 + 3t\dot{x}\ .$$

V4.12.2. Find all extremals for the following classical variational problem

V4.2.12.

$$\mathcal{J}[x(t)] = \int_0^2 (t^2 + \dot{x}^2 - x^2)\, dt \ \to \text{extr}\ , \quad x(0) = 0\,,\ x(2) = \sin 2\ .$$

V4.12.3. Find the extremals for the Bolza problem

$$\mathcal{B}[x(t)] = \int_0^{e-1} (t+1)\dot{x}^2\, dt + 2x(0) + 2x(0)\, x(e-1) \ \to \text{extr}\ \ .$$

V4.12.4. Find the extremals for the isoperimetric problem.

$$\mathcal{J}_0[x(t)] = \int_1^2 t\,\dot{x}^2\, dt \ \to \text{extr}\ , \quad \int_1^2 tx\, dt = 21/4\ ,\ x(1) = 2\ ,\ x(2) = 5\,.$$